

On the Limitations of Wilson's Renormalization Group Program

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Abstract

This informal report surveys several lesser-known limitations of Wilson's Renormalization Group program. The account is not intended to be either rigorous or complete as our sole purpose is to stimulate further discussions and research.

Key words: Perturbative Renormalization Group, Renormalization Group flow, Effective Field Theory, Standard Model, Fixed Points, Fractal Spacetime, Minimal Fractal Manifold.

It is widely recognized that a key program built in the structure of the Standard Model of high-energy physics (SM) is the Renormalization Group (RG), whose function is to preserve self-consistency and describe how parameters of the theory evolve with the energy scale. The Wilson treatment of critical phenomena using the perturbative *Renormalization Group* program (RG) [1, 11] develops from the premise that quantum fields present in the theory (Φ_μ) depend on the running scale μ and can be segregated into a pair of un-coupled components

$$\Phi_\mu^{(l)} : 0 \leq \mu \leq \frac{\Lambda_{UV}}{s} \quad (1)$$

$$\Phi_\mu^{(s)} : \frac{\Lambda_{UV}}{s} \leq \mu \leq \Lambda_{UV} \quad (2)$$

Here, Λ_{UV} stands for the cutoff scale in the ultraviolet sector, the parameter “ s ” is an arbitrary scaling factor ($s > 1$), $\Phi_{\mu}^{(l)}$ and $\Phi_{\mu}^{(s)}$ are the long and short wavelength excitations and correspond, respectively, to the light and heavy particles carried by Φ_{μ} . Starting with an effective field theory (EFT) defined at Λ_{UV} , the core idea of Wilson’s approach is to integrate out all heavy particles contained in the “momentum shell” (2) and form a new EFT with the remaining fields below the separation scale Λ_{UV}/s . Since μ is considered a running parameter, iterating this process yields a flow of EFT’s from Λ_{UV} toward their low-energy limit. It is customary to refer to this iterative process as a RG flow (or RG trajectory). A key property of local EFT’s is that the low-energy endpoint of the RG flow must describe phenomena that are fully decoupled from physical processes occurring near the high-energy limit Λ_{UV} . This property conveys the basic idea behind the concept of *scale invariance* [10].

Despite being accepted as a paradigm for securing consistency of the SM, Wilson’s renormalization model presents several lesser-known aspects and limitations which are often neglected in standard textbooks. Namely,

- The RG flow is *not laminar* in general, turbulent behavior is a possibility that cannot be excluded [1-2, 4-5].
- The hypothesis of *local* (next-neighbor) coupling of fields across the RG flow may be violated in the presence of un-damped excitations perturbing the conventional “smooth” evolution towards fixed points.
- The RG flow may display complex structure and non-trivial dynamics near fixed points, including *ergodic* behavior, *limit cycles* and *strange attractors* [1, 5-7, 12].

- Decoupling of long and short wavelength excitations (1) and (2) may not be possible in general when *non-local* couplings come into play. Typical examples are RG flows with temporal memory (non-Markovian flows) or long-range spatial interactions that may surface in the mid to the deep TeV region of high-energy physics [13-14].
- The RG flow is neither *linear* nor *perturbative* in general [2].
- Under sizable deviations from four dimensions $\varepsilon = 4 - D$, $\varepsilon \sim O(1)$, the epsilon expansion advocated by the Wilson model leads to the emergence of *negative norm states* [3]. Likewise, Lorentz symmetry turns out to be ill-defined as a result of the intrinsic non-differentiability of fractal trajectories [9-10]. The condition $\varepsilon \ll 1$, defined within the framework of the so-called *minimal fractal manifold* (MFM), is the only sensible setting where fractal geometry asymptotically approaches all consistency requirements mandated by EFT and the SM [8-10].

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